Lecture #1: Robustness and 1st generation CRONE control design
Contents

1 – Robustness
   1.1-Stability degree and stability margins
   1.2-Robustness and stability margins

2 - CRONE Control Design
   2.1-First generation
   2.2-Second generation
   2.3-Third generation
   2.4-Specific problems
   2.5-MIMO Systems
   2.6- Industrial applications

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User's forum:

3 – QFT Design
   3.1-Principle and methodology
   3.2-Application examples
In the Automatic Control field,

the robustness is the ability of a control-system to ensure an almost constant closed-loop characteristic,

and more particularly to ensure small variations of the closed-loop system stability-degree

although the controlled plant is perturbed and its model uncertain.
When the Nyquist stability condition is met, for a SISO case, the closed-loop stability-degree is often estimated from the "distance" between the open-loop Nyquist plot and the −1 point measured using stability margins:

- **phase margin** $M_\phi$
- **gain margin** $M_G$
- **time-delay margin** $M_\theta$
- **modulus margin** $M_m$

\[
M_\theta = \min_{\omega_{cg}} \left[ 180^\circ + \arg \beta(j\omega_{cg}) \right]
\]
Stability degree and stability margins 2/4

The closed-loop stability-degree can also be estimated directly from time-domain closed-loop features:

- percentage overshoot $O\%$ of the step response
- damping ratio $\zeta$ related to the dominant closed-loop pole pair if it exists

$$\zeta = \cos \frac{\theta}{2}$$

$$\frac{(y(t)-y(0))/(y(\infty)-y(0))}{(y(t)-y(0))/(y(\infty)-y(0))}$$
Stability degree and stability margins 3/4

The closed-loop stability-degree can also be estimated directly from frequency-domain closed-loop features:

- **Resonant peak $M_r$ of $T(j\omega) = \frac{\beta(j\omega)}{1 + \beta(j\omega)}$**

- **Peak value related to the sensitivity function $S(s)$**

$$S(s) = \frac{\partial T(s)}{T(s)} \left/ \frac{\partial G(s)}{G(s)} \right. = \frac{1}{1 + \beta(s)} = 1 - T(s)$$

$$\max_\omega |S(j\omega)| = \frac{1}{\min_\omega |1 + \beta(j\omega)|} = \frac{1}{M_m}$$

$M_r = 3 \text{dB}$

Frequency (rad/sec)

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Resonant peak $M_r$ is (often) highly correlated to overshoot $O\%$. $M_r$ can be determined (or set) using the magnitude-contours of the Nichols plot.

$$T(j\omega) = \frac{\beta(j\omega)}{1 + \beta(j\omega)}$$
The final goal is the robustness of time-domain characteristics: overshoot $O\%$ and/or damping ratio $\zeta$.

Nevertheless, in a frequency-domain control-system design, the stability-degree robustness can be ensured by minimizing the variation in the phase margin $M_\phi$ or, more reliably, in the resonant peak $M_r$.

In this non-robust case, when the plant is perturbed:

- phase margin $M_\phi$ can vary from $29^\circ$ to $52^\circ$
- resonant peak $M_r$ can vary from 2dB to 6dB
CRONE is a French acronym which means: fractional order robust control

- Frequency-domain based methodology using fractional differentiation as high-level design parameter (since 1975)
- Continuous or discrete time control of perturbed SISO and MIMO systems
- Use of the common unity-feedback configuration
- Robustness of the stability-degree with respect to the parametric plant perturbation (no over-estimation)
- Avoiding over-estimation of plant perturbation leads to non-conservative robust control-systems and to performance as good as possible
- Control of minimum or non-minimum phase plants, unstable plants or plants with bending modes, time-varying plants, nonlinear plants
CRONE control (3 generations)

3 CRONE design generations have been developed, successively extending the application fields:

- 1\textsuperscript{st} for gain-like plant perturbation model and for constant plant phase around $\omega_{cg}$ (real fractional diff. order for controller definition)
- 2\textsuperscript{nd} for gain-like plant perturbation model (real fractional diff. order for open-loop definition)
- 3\textsuperscript{rd} for most general perturbation model (complex fractional diff. order(s) for open-loop definition)
The CRONE controller is defined within a frequency range \([\omega_A, \omega_B]\) around the desired open-loop gain-crossover frequency \(\omega_{cg}\) from the fractional transfer function of an order \(n\) integro-differentiator:

\[
C_F(s) = C_0 s^n, \text{ with } n \text{ and } C_0 \in \mathbb{R}.
\]

The constant phase \(n\pi/2\) characterizes this controller around frequency \(\omega_{cg}\). When frequency \(\omega_{cg}\) varies, the constant phase controller does not contribute to the phase margin variations.
First generation (robustness)

Particularly appropriate when the desired open-loop gain crossover frequency $\omega_{cg}$ is within a frequency range where the plant frequency response is asymptotic.

- Close to $\omega_{cg}$, the plant uncertainty is only gain-like: variation of plant corner-frequencies greatly different from $\omega_{cg}$ or/and plant gain variation
- The frequency range $[\omega_A, \omega_B]$ must at least equal the range where frequency $\omega_{cg}$ can vary
- The phase margin $M_\phi$ equals $(n+p+2)\pi/2$
First generation (rational version)

Around $\omega_{cg}$, the initial fractional version $C_F(s)$ of the controller can also be defined by a band-limited transfer function using corner frequencies:

$$C_F(s) = C_0 \left( \frac{1 + s/\omega_1}{1 + s/\omega_h} \right)^n$$

with $\omega_1 < \omega_A$ and $\omega_h > \omega_B$.

Achievable rational version $C_R(s)$ of the controller, which can be implemented, defined by a transfer function resulting from of a recursive distribution of $N$ cells of real negative zeros and poles:

$$C_R(s) = C_0 \prod_{i=1}^{N} \frac{1 + s/\omega'_i}{1 + s/\omega_i}$$

with $N \in \mathbb{N}^+$ and $\omega'_i$, $\omega_i \in \mathbb{R}^+$.

$n = \log \alpha / \log \alpha . \eta$
To manage the control effort level and steady state errors, $C_F$ (and thus $C_R$) has to be complexified. $C_F(s)$ needs to include:

- an order $n_I$ band-limited integrator
- an order $n_F$ low-pass filter.

$$C_F(s) = C_0 \left( \frac{\omega_I}{s} + 1 \right)^{n_I} \left( \frac{1 + s/\omega_1}{1 + s/\omega_h} \right)^n \frac{1}{(1 + s/\omega_F)^{n_F}}$$

with $n_I, n_F \in \mathbb{N}^+$, and $\omega_1, \omega_F \in \mathbb{R}^+$.

- $\omega \in [0, \omega_1]$ : Integral effect
- $\omega \in [\omega_1, \omega_h]$ : Proportional effect
- $\omega \in [\omega_h, \omega_F]$ : order $n$ fract. Diff. effect
- $\omega \in [\omega_F, \infty[$ : HF gain effect
- $\omega \in [\omega_F, \infty[$ : low-pass effect

This is a $\text{PID}^n$ controller
Let a plant be modelled by a LTI model

\[ G(s) = \frac{y(s)}{u(s)} = \frac{k}{s(1 + \tau s)} \]

\[ k = 10 \]
\[ \tau_0 = 1000\text{s} \]
\[ \tau_0/9 \leq \tau \leq \tau_0*9 \]

\[ d_u \text{ low-frequency disturbance} \]
\[ N_m \text{ high-frequency measurement noise} \]

For all \( \tau \), we want to:
- limit the control-effort \( u \) due to the measurement noise
- reduce the steady-state effect of a step disturbance \( d_u \) to zero with a settling time as short as possible
- obtain a step response of \( y \) to a reference input \( y_{\text{ref}} \)
  - with a percentage overshoot around 25%
  - and a settling time as short as possible

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First generation (application 2/6)

\[ G(s) = \frac{k}{s(1 + \tau s)} \quad k = 10, \ \tau_0 = 1000s, \ \tau_0/9 \leq \tau \leq \tau_0/9 \]

- Let us consider that the control effort limitation leads to a nominal value of \( \omega_{cg} = 5 \) rad/s.
- A phase margin of 50° leads to an overshoot about 25%.
- The constant open-loop phase around \( \omega_{cg} \) will equal \(-130° \) (-180°+50°=-1.44*90°).
- The constant open-loop gain slope around \( \omega_{cg} \) will equal -28.8dB/dec.
- As the plant gain variations around \( \omega_{cg} \) equals 38.2dB, the frequency band \([\omega_A, \omega_B]\) needs at least to cover 1.32 decade.
- Then \( \omega_B/\omega_A = 22 \) leads to \( \omega_A = 1.07 \) rad/s and \( \omega_B = 23.5 \) rad/s.

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First generation (application 3/6)

- To reject the input disturbance $d_u(t)$, $n_I = 1$ and we set $\omega_I = \omega_{cg}/200$
- To avoid the amplification of the high-frequency measurement noise, $n_F = 1$ and we set $\omega_F = \omega_{cg} * 200$
- Avoiding damage to the constant phase leads to $\omega_I = \omega_A/10$ and $\omega_h = \omega_B * 10$
- $n = 0.577$ ensures that $\arg(C_F(j5)G_0(j5)) = -130^\circ$
- $C_0 = 271$ ensures that $|C_F(j5)G_0(j5)| = 1$.

\[
C_F(s) = C_0 \left( \frac{\omega_I}{s} + 1 \right) \left( \frac{1 + s/\omega_1}{1 + s/\omega_h} \right)^n \frac{1}{(1 + s/\omega_F)}
\]

For common PID design $n = 1$
- Now, $\omega_I = \omega_{cg}/2.79$ and $\omega_h = \omega_{cg} * 2.79$ ensure that $\arg(C_{PID}(j5)G_0(j5)) = -130^\circ$
- $C_0 = 896$ for $|C_{PID}(j5)G_0(j5)| = 1$.

\[
C_{PID}(s) = C_0 \left( \frac{\omega_I}{s} + 1 \right) \left( \frac{1 + s/\omega_1}{1 + s/\omega_h} \right) \frac{1}{(1 + s/\omega_F)}
\]
First generation (application 4/6)

PID controller:
- $25^\circ \leq M_\phi \leq 50^\circ$
- $2.6\text{dB} \leq M_r \leq 7.5\text{dB}$

CRONE controller:
- $48^\circ \leq M_\phi \leq 50^\circ$
- $2.3\text{dB} \leq M_r \leq 2.8\text{dB}$
Fractional part (\( n = 0.577 \)) of the CRONE controller synthesised using \( N = 5 \) pole-zero cells.

With \( n = \log \alpha / \log(\alpha \eta) \)

and \( (\alpha \eta)^N = \frac{\omega_h}{\omega_1} \)

\( \eta = 1.916 \) and \( \alpha = 2.432 \)

\[
C_F(s) = C_0 \left( \frac{\omega_1}{s} + 1 \right) \left( \frac{1 + s/\omega_1}{1 + s/\omega_h} \right)^n \frac{1}{(1 + s/\omega_F)}
\]

\[
C_R(s) = C_0 \left( \frac{\omega_1}{s} + 1 \right)^N \prod_{i=1}^{N} \frac{1 + s/\omega_1}{1 + s/\omega_i} \frac{1}{(1 + s/\omega_F)}
\]
First generation (application 6/6)

PID controller: $28\% \leq 0_\% \leq 56\%$

CRONE controller: $25\% \leq 0_\% \leq 30\%$

CRONE Vs. PID: one parameter more to manage robustness
Due to control effort limitation specification (or high input sensitivity function magnitude), it is sometimes impossible to choose an open-loop gain crossover frequency within an asymptotic behavior frequency band of the plant.

Thus when the desired $\omega_{cg}$ is outside an asymptotic behavior band, the first generation CRONE controller can not ensure the robustness of the closed-loop system stability margins.

Nevertheless, as Bode first stated (Network Analysis and Feedback Amplifier Design, Van Nostrand, New York, 1945):

- for the “design of single loop absolutely stable amplifiers” whose tube gains vary,
- the robust controller is the one which ensures an open-loop transfer function defined by a constant phase (and constant dB/oct gain slope) in a useful band.
Electrical Engineering Department
Semester 9 AM2AS

UE E9AM2AS-C AU306
Frequency-domain design of robust control-systems

Lectures #2&3: 2nd and 3rd generation CRONE control designs
Contents

1 - Robustness
   1.1-Stability degree and stability margins
   1.2-Robustness and stability margins

2 - CRONE Control Design
   2.1-First generation
   2.2-Second generation
   2.3-Third generation
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First generation limitation – Bode’s early principle

Due to control effort limitation specification (or high input sensitivity function magnitude), it is sometimes impossible to choose an open-loop gain crossover frequency within an asymptotic behavior frequency band of the plant.

Thus when the desired $\omega_c$ is outside an asymptotic behavior band, the first generation CRONE controller can not ensure the robustness of the closed-loop system stability margins.


- for the “design of single loop absolutely stable amplifiers” whose tube gains vary,
- the robust controller is the one which ensures an open-loop transfer function defined by a constant phase (and constant dB/oct gain slope) in a useful band.
2.2-Second generation CRONE control

When $\omega_{cg}$ is within a frequency band where the plant uncertainties are gain-like, the CRONE approach defines the open-loop transfer function (in the frequency band $[\omega_A, \omega_B]$) by that of a fractional integrator:

$$T(s) = \frac{\beta(s)}{1 + \beta(s)} = \frac{1}{1 + \left(\frac{s}{\omega_{cg}}\right)^n}$$

$$\beta(s) = \left(\frac{\omega_{cg}}{s}\right)^n$$, with $n \in \mathbb{R}$ and $n \in [1,2]$.

This vertical straight line being the desired shape of the open-loop Nichols plot, we call it a frequency template or simply template.

**Remark:** $G(s)$ is minimum phase

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Second generation (robustness)

At the time of plant perturbation, the vertical displacement of the vertical template ensures the robustness of:

• the phase margin $M_\phi = (2-n)\pi/2$

$$\sup_{\omega} |T(j\omega)| = \frac{1}{\sin(n \pi/2)}$$

• the resonant peak $M_r = \frac{\omega}{|T(j0)|} = 1$

• the modulus margin $M_m$

$$M_m = \inf_{\omega} |\beta(j\omega) + 1| = \left(\sup_{\omega} |S(j\omega)|\right)^{-1} = \sin(n \pi/2)$$

• the damping ratio $\zeta$ (related to the closed-loop poles)

$$\zeta = -\cos(\pi/n)$$
Second generation (parameters)

To manage the control effort level and the steady-state errors, the fractional open-loop transfer function has to be band-limited and to include integral and low-pass effects:

\[
\beta(s) = K \left( \frac{\omega'_1}{s} + 1 \right)^{n_l} \left( \frac{1 + s/\omega_h}{1 + s/\omega_l} \right)^n \frac{1}{\left(1 + s/\omega'_h\right)^{n_h}}
\]

with \(\omega'_1, \omega_l, \omega_h, \omega'_h\) and \(K \in \mathbb{R}^+, n_l \) and \(n_h \in \mathbb{N}^+\).

Integer orders \(n_l\) and \(n_h\) are set by taking into account the performance specifications and the plant magnitude asymptotic behavior at low and high frequencies.
Second generation (parameters)

\[ \beta(s) = K \left( \frac{\omega'_1}{s} + 1 \right)^{n_l} \left( \frac{1 + \frac{s}{\omega_h}}{1 + \frac{s}{\omega'_1}} \right)^{n} \frac{1}{\left( 1 + \frac{s}{\omega'_h} \right)^{n_h}} \]

- As usually: \( \omega'_1 \leq \omega_1 < \omega_{cg} < \omega_h \leq \omega'_h \)

- As \( n_{pl} \) and \( n_{ph} \) represents the order of the plant magnitude asymptotic behavior at low frequency (\( \omega < \omega'_1 \)) and at high frequency (\( \omega > \omega'_h \)), order \( n_1 \) and \( n_h \) are given by
  \[ n_1 \geq n_{pl} \quad \text{and} \quad n_h \geq n_{ph} \]

\( \omega'_1 = \omega_1 \) and \( \omega_h = \omega'_h \)

\[ n_1 = 2 \quad \text{and} \quad n_h = 3 \]
Second generation (controller synthesis)

2 ways to synthesize the rational form of the controller

• From its ideal frequency response

\[ C(j\omega) = \frac{\beta(j\omega)}{G_0(j\omega)} \]

identification of \( C_F(j\omega) \) by a low-order transfer function \( C_R(s) \)

\[ C_R(s) = \frac{C_0}{s^{N_{int}}} \prod_{i=1}^{n_1} \left( \frac{1}{1 + \frac{s}{\omega_{z_i}}} \right) \prod_{i=1}^{n_2} \left( \frac{1}{1 + \frac{s}{\omega_{n_{z_i}}}} \right) \prod_{i=1}^{d_1} \left( \frac{1}{1 + \frac{s}{\omega_{p_i}}} \right) \prod_{i=1}^{d_2} \left( \frac{1}{1 + \frac{s}{\omega_{n_{p_i}}}} \right) \]

(use of a frequency-domain system-identification method)

• Using the rational approximation of \( \beta \)

\[ \beta_R(s) = K \left( \frac{\omega_1'}{s} + 1 \right)^{n_1} \prod_{i=1}^{N} \frac{1 + s/\omega_i'}{1 + s/\omega_i} \left( \frac{1}{1 + s/\omega_h} \right)^{n_h} \]

\[ \text{and} \quad C_R(s) = G_0^{-1}(s)\beta_R(s) \]

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The previous model is modified to include "high-frequency" dynamics:

\[ G(s) = \frac{10}{s(1+\tau s)} \left( \frac{1+s}{\omega_z} \right) \left( 1+\frac{s}{\omega_p} \right) \left( 1+\frac{s}{10} \right) \left( 1+\frac{s}{100} \right) \]

\[ \tau_0 = 1000s \quad \text{and} \quad \frac{\tau_0}{3} \leq \tau \leq \tau_0 \times 3 \]

\[ \omega_p = 1 \text{ rad/s} \quad \text{and} \quad \omega_z = 50 \text{ rad/s} \]

As the specifications remain the same, the nominal value of \( \omega_{cg} \) will be 5 rad/s with a phase margin of 50°.
Second generation (application 2/5)

\[ \beta(s) = K \left( \frac{\omega_1}{s} + 1 \right)^{n_l} \left( \frac{1}{1 + \frac{s}{\omega_h}} \right)^n \left( 1 + \frac{s}{\omega_l} \right)^{n_h} \]

- The constant open-loop phase around \( \omega_{cg} \) will equal \(-130^\circ\)
- The constant open-loop gain slope around \( \omega_{cg} \) will equal \(-28.8\text{dB/dec}\)
- As the plant gain variations around \( \omega_{cg} \) equals 19.1\text{dB}, the frequency band \([\omega_A, \omega_B]\) needs at least to cover 0.66 decade (\(\omega_B/\omega_A = 5\))
- With \(\omega_B/\omega_A = 10\) leads to \(\omega_A = 1.58\text{ rad/s} \) and \(\omega_B = 15.8\text{ rad/s}\)
- To reject the input disturbance, \(n_l = 3\)
- To avoid the amplification of the measurement noise, \(n_h = 5\)
- Taking into account side effects lead to set \(\omega_l = \omega_A/20\) and \(\omega_h = \omega_B*40\)

- \(n = 1.41\) ensures \(\text{arg}(\beta(j5)) = -130^\circ\)
- \(K = 347\) ensures \(|\beta(j5)| = 1\)
\[ 49^\circ \leq M_p \leq 50^\circ \]

\[ 2.3 \text{dB} \leq M_r \leq 2.5 \text{dB} \text{ and } 0.764 \leq M_m \leq 0.768 \]
Second generation (application 4/5)

Fractional part of \( n = 1.41 \) of \( \beta_R(s) \) approximated using \( N = 5 \) pole-zero cells.

With \( 0.41 = \log \alpha / \log(\alpha \eta) \)

and \( (\alpha \eta)^N = \frac{\omega_h}{\omega_1} \)

\( \eta = 2.89 \) and \( \alpha = 2.09 \)

\[
\beta(s) = K \left( \frac{\omega_1}{s} + 1 \right)^{n_1} \left( \frac{1 + s/\omega_h}{1 + s/\omega_1} \right)^n \frac{1}{\left( 1 + s/\omega_h \right)^{n_h}}
\]

\[
\beta_R(s) = K \left( \frac{\omega_1}{s} + 1 \right)^{n_1} \prod_{i=1}^{N} \frac{1 + s/\omega_i}{1 + s/\alpha i} \frac{1}{\left( 1 + s/\omega_h \right)^{n_h}}
\]

\[
C_R(s) = G^{-1}_0(s) \beta_R(s)
\]
Second generation (application 5/5)

Perfect robustness in spite of a gain variation of a factor 9

25% ≤ 0% ≤ 27%

Plant output y(t)

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Second generation (phase variations 1/3)

The previous model is used again with new parameter values:

\[ G(s) = \frac{10}{s(1 + \tau s)} \left( \frac{1 + \frac{s}{\omega_z}} {1 + \frac{s}{\omega_p}} \right) \left( 1 + \frac{s}{10} \right) \left( 1 + \frac{s}{100} \right) \]

\( \tau = 1000s \) and \( \omega_z = 50 \text{ rad/s} \)

\( \omega_{p0} = 1 \text{ rad/s} \) and \( \omega_{p0}/3 \leq \omega_p \leq \omega_{p0} \times 3 \)

Around \( \omega = 5 \text{ rad/s} \), the frequency response not only exhibits gain variation.

What happens if we use the previous second generation CRONE controller?
Second generation (phase variations 2/3)

\[ 34^\circ \leq M_\phi \leq 59^\circ \]

\[ 0.74\text{dB} \leq M_r \leq 5.4\text{dB} \]

\[ 0.76 \leq M_m \leq 0.81 \]

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Second generation (phase variations 3/3)

The vertical template does not provide a good robustness at the time of not only gain-like plant uncertainty

$12\% \leq 0\% \leq 45\%$
Second generation (extension 1/2)

Is it possible to obtain a best robustness?

![Graph showing plant output y(t) over time](image)

\[20\% \leq 0_{\%} \leq 28\%\]

Of course, it is possible ...

... but how?
Second generation (extension 2/2)

Idea: replacement of the vertical template by a template with the same direction as that of the uncertainty domains around $\omega_r$

$M_r$ up to 4dB

$M_r$ up to 2.5dB only

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2.3-Third generation CRONE control

We considers now the template as:

- always defined as a straight line segment for the nominal parametric state of the plant;
- but which does not necessarily determine a constant phase around frequency $\omega_{cg}$.

The vertical straight line segment used to define the template in the Nichols plane, is thus replaced by a line segment of any-angle, called a generalized template.
Third generation (generalized template)

Whereas the vertical template is based on the real fractional-order transfer function

\[ \beta(s) = \left( \frac{\omega_{cg}}{s} \right)^n, \text{ with } n \in \mathbb{R} \]

this generalized template is based on the real part (with respect to imaginary unit number denoted “i”) of the fractional complex integration:

\[ \beta(s) = \left[ \cosh \left( b \frac{\pi}{2} \right) \right]^{\text{sign}(b)} \left( \frac{\omega_{cg}}{s} \right)^a \left( \mathcal{R}_{/i} \left[ \left( \frac{\omega_{cg}}{s} \right)^i b \right] \right)^{\text{sign}(b)} \]

with \( n = a + i b \in \mathbb{C}_i \) and \( s = \sigma + j \omega \in \mathbb{C}_j \)

or

\[ \beta(s) = \left[ \cosh \left( b \frac{\pi}{2} \right) \right]^{\text{sign}(b)} \left( \frac{\omega_{cg}}{s} \right)^a \left[ \cos \left( b \ln \left( \frac{s}{\omega_{cg}} \right) \right) \right]^{\text{sign}(b)} \]
Third generation (\(a\) and \(b\) effects)

In the frequency range \([\omega_A, \omega_B]\), \(\beta(j\omega) = \left[ \cosh\left( b \frac{\pi}{2} \right) \right] \text{sign}(b) \left( \frac{\omega_{cg}}{j\omega} \right)^a \left[ \cos \left( b \ln \left( \frac{j\omega}{\omega_{cg}} \right) \right) \right]\)^{-\text{sign}(b)}

In the Nichols chart at frequency \(\omega_{cg}\), the real order \(a\) determines the phase location of the template, \(a\pi/2\), and the imaginary order \(b\) then determines its angle to the vertical.
CRONE control deals with the robustness of both stability margins and performance, and particularly the robustness of resonant peak $M_r$. Let $M_{r0}$ be the required magnitude peak of $M_r$ for the nominal plant $G_0$.

An indefinite number of open-loop Nichols loci can tangent the $M_{r0}$ M-contour. Also, for perturbed plants, parametric variations lead to variations of $M_r$.

Thus, a generalized template is defined as optimal if it tangents the $M_{r0}$ M-contour around $\omega_r$ for the nominal plant state, and if it minimizes the variations of $M_r$ for the other plant states.
Let an uncertain LTI SISO plant be:
\[ G(s) = G_0(s)\Delta G(s) \]

The uncertainty domains related to the Nichols plot \( G_0(j\omega) \) are defined by all the possible values of the pair \( \{ |\Delta G(j\omega)|_{dB}, \arg\Delta G(j\omega) \} \).

The perturbed open-loop transfer function is defined by:
\[ \beta(s) = C(s)G(s) = C(s)G_0(s)\Delta G(s) = \beta_0(s)\Delta G(s). \]

Thus, the uncertainty domains related to the Nichols plot of \( \beta_0(j\omega) \) are also defined by the possible values of the pair \( \{ |\Delta G(j\omega)|_{dB}, \arg\Delta G(j\omega) \} \).

The nominal open-loop frequency response can only move the uncertainty domains \( D \) (in the Nichols chart) through longitudinal and vertical translations.
The optimal template positions the open-loop uncertainty domains correctly, so that they overlap the low stability margin areas as little as possible.

**Minimization of $M_r$ variations**

Generalized template $\rightarrow$ Optimal template

$M_{r\ max1} - M_{r0} \gg M_{r\ max2} - M_{r0}$
To manage the control effort level and the steady-state errors, the fractional open-loop transfer function has to be band-limited and complexified by including integral and low-pass effects:

$$
\beta(s) = K \left( \frac{\omega_1}{s} + 1 \right)^{n_l} \left( \frac{1 + \frac{s}{\omega_h}}{1 + \frac{s}{\omega_l}} \right)^{a} \Re \left[ \frac{C_0}{1 + \frac{s}{\omega_h}} \right]^{i b} \frac{1}{1 + \frac{s}{\omega_h}}^{n_h}
$$

with

$$
C_0 = \left( 1 + \left( \frac{\omega_r}{\omega_l} \right)^2 \right) / \left( 1 + \left( \frac{\omega_r}{\omega_h} \right)^2 \right)^{1/2}
$$

and with $\omega_1$, $\omega_h$ and $K \in \mathbb{R}^+$, $n_l$ and $n_h \in \mathbb{N}^+$. 
$n_1, n_h, a, b, \omega_l, \omega_h, \omega_r$ and $K$ are 8 high-level parameters of the open-loop transfer function $\beta(s)$.

As the control-system designer set $n_1$ and $n_h$ (cf 2nd generation), and as a tangency condition ($M_{r0}$) is required (thus 2 parameters will be dependent), only 4 independent parameters have to be optimized.

Each parameter acts only on one shape feature of $\beta(j\omega)$, and thus can be optimized easily.

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A nonlinear optimization algorithm searches for the vector of 4 independent parameters which:

- minimizes a cost function $J$ based on resonant peak variations: $\max G M_{\text{rdB}} - M_{\text{r0dB}}$
- fulfils a set of 5 shaping constraints on the 4 usual sensitivity functions $T(s)$, $S(s)$, $CS(s)$ and $SG(s)$.

The 4 independent high-level parameters optimized are often:

- corner frequency $\omega_l$ and $\omega_h$,
- resonant frequency $\omega_r$,
- tangency point (defined by $Y_r$)
To manage precisely performance related to tracking, regulation and control effort level, 5 inequality constraints are to be fulfilled for all plants (or parametric states of the plant) and for \( \omega \in \mathbb{R}^+ \):

\[
\begin{align*}
\inf_G |T(j\omega)| & \geq T_1(\omega) \quad \text{and} \quad \sup_G |T(j\omega)| \leq T_u(\omega) \\
\sup_G |S(j\omega)| & \leq S_u(\omega) \\
\sup_G |CS(j\omega)| & \leq CS_u(\omega) \\
\sup_G |GS(j\omega)| & \leq GS_u(\omega)
\end{align*}
\]

\[
T(s) = \frac{Y(s)}{Y_{\text{ref}}(p)} - \frac{Y(s)}{N_m(s)} - \frac{U(s)}{D_u(s)} = \frac{C(s)G(s)}{1+C(s)G(s)}
\]

\[
S(s) = \frac{Y(s)}{D_y(s)} = \frac{1}{1+C(s)G(s)}
\]

\[
GS(s) = \frac{Y(s)}{D_u(s)} = \frac{G(s)}{1+C(s)G(s)}
\]

\[
SC(s) = \frac{U(s)}{Y_{\text{ref}}(s)} = -\frac{U(s)}{N_m(s)} = -\frac{U(s)}{D_y(s)} = \frac{C(s)}{1+C(s)G(s)}
\]
Third generation (shaping constraints 2/2)

- **T magnitude (dB)**
  - limit the sluggishness
  - limits the highest resonant peak
  - limits the measurement noise effect

- **S magnitude (dB)**
  - limits the lowest modulus margin
  - improves the output disturbance rejection

- **CS magnitude (dB)**
  - limits the lowest cutoff frequency
  - limits the control effort

- **SG magnitude (dB)**
  - improves the input disturbance rejection

Frequency (rad/s)
Its is sometime usefull to have more than 4 parameters that can be tuned. Based on a multi-template, the transfert function is defined by:

$$\beta(s) = K \left( \frac{\omega_{-N^-}}{s} + 1 \right)^{n_1} \prod_{k=-N^-}^{N^+} \left( \frac{1 + s/\omega_{k+1}}{1 + s/\omega_k} \right)^{a_k} \left( \alpha_k \frac{1 + s/\omega_{k+1}}{1 + s/\omega_k} \right)^{ib_k} \right)^{-q_k \text{sign}(b_k)} \left( 1 + s / \omega_{N^++1} \right)^{n_h}$$

with $$\alpha_0 = \left( 1 + \frac{\omega_r}{\omega_0} \right)^2 \left/ \left( 1 + \frac{\omega_r}{\omega_1} \right)^2 \right)^{1/2}$$

$$\alpha_k = \left( \omega_{k+1}/\omega_k \right)^{1/2} \text{ for } k \neq 0$$

and $$\omega_k \text{ and } K \in \mathbb{R}^+$$

$$a_k \text{ and } b_k \in \mathbb{R}$$

$$n_1 \text{ and } n_h \in \mathbb{N}^+.$$
Third generation (curvilinear template 2/2)

The genuine uncertainty domains being used, a non-linear optimization method must be used to find the optimal values of the independent parameters.

The parameterization of the open-loop transfer function by complex fractional order of integration, then simplifies the optimization considerably (during optimization the complex order has, alone, the same function as many parameters found in common rational controllers)

The introduction of a further generalized template leads to 3 further independent parameters. Thus, $N^+$ and $N^-$ have to be as small as possible to don't loose the advantage of complex integration which permits parameterization with few parameters.

$$
\beta(s) = K \left( \frac{\omega^{N^-}}{s} + 1 \right)^{m_1} \prod_{k=-N^-}^{N^+} \left[ \frac{1 + s/\omega_{k+1}}{1 + s/\omega_k} \right] a_k \left\{ \Re \left\{ \alpha_k \left( \frac{1 + s/\omega_{k+1}}{1 + s/\omega_k} \right)^{i b_k} \right\} \right\}^{-q_k \text{sign} (b_k)} \left[ \frac{1}{1 + s / \omega_{N^+ + 1}} \right]^{n_h}
$$
The plant is the previous one:

\[ G(s) = \frac{10}{s(1+\tau s)} \left( \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \right) \left( \frac{1}{1 + \frac{s}{10}} \frac{1}{1 + \frac{s}{100}} \right) \]

\[ \tau = 1000 \text{s and } \omega_z = 50 \text{ rad/s} \]
\[ \omega_p = 1 \text{ rad/s and } \omega_p / 3 \leq \omega_p \leq \omega_p \times 3 \]

• The required nominal resonant peak is 2 dB
• The designer set: \( n_1 = 3, n_h = 5 \) and \( N^+ = N^- = 0 \)

\[ \beta(s) = K \left( \frac{\omega_0}{s} + 1 \right)^3 \left( \frac{1 + s/\omega_1}{1 + s/\omega_0} \right)^{a_0} \text{Re} \left[ \left( \frac{\omega_0}{1 + s/\omega_0} \right)^{ib_0} \right]^{\text{-sign}(b_0)} \frac{1}{(1 + s/\omega_1)^5} \]

• The optimization leads to \( \omega_r = 4.55 \text{ rad/s}, Y_0 = 1.83 \text{dB}, \omega_0 = 0.081 \text{ rad/s} \) and \( \omega_1 = 317 \text{ rad/s} \).

• Then \( a_0 = 1.38, b_0 = -0.47 \) and \( K = 255 \).
Third generation (application 2/5)

$M_r \leq 2.5 \text{ dB}$

$M_m \geq 0.58$
The five constraints are fulfilled
An order-6 rational system is used to synthesize the rational controller which fits the desired frequency response of the controller.
As the plant perturbations taken into account are the genuine one, the linear robust controller is as good as possible (no conservatism)

$$20\% \leq \delta_c \leq 28\%$$

Now you know ...
Improvement of the first generation CSD (1/3)

We want to modify the initial first generation CSD to achieve the robustness at the time of:

• a non constant phase of the plant along the frequency range where \( \omega_{cg} \) varies
• non gain-like uncertainty around \( \omega_{cg} \)

The main idea is to design a controller that provides an open loop with a given behavior around \( \omega_{cg} \):

\[
\arg \beta(j\omega) = \arg \beta(j\omega_{cg}) + A \log \frac{\omega}{\omega_{cg}}
\]

If around \( \omega_{cg} \) the plant is such as

\[
\arg G(j\omega) = \arg G(j\omega_{cg}) + B \log \frac{\omega}{\omega_{cg}}
\]

around \( \omega_{cg} \), the controller needs to ensure:

\[
\arg C(j\omega) = \arg \beta(j\omega_{cg}) - \arg G(j\omega_{cg}) + (A - B) \log \frac{\omega}{\omega_{cg}}
\]

Constant phase controller
Logarithmic phase controller

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Improvement of the first generation CSD (2/3)

As the phase of $G$ is not logarithmic within a wide frequency range, this strategy is useful only when $\omega_{cg}$ varies weakly. Then the constant and logarithmic phase controllers can be synthesized both with only 2 pole/zero couples each.

$$\alpha = \eta \frac{1}{2} \tan \left( \frac{1}{2} \arg C(j \omega_{cg}) + \arctan \eta \frac{1}{2} \right)$$

with $\arg C(j \omega_{cg}) = \arg \beta(j \omega_{cg}) - \arg G(j \omega_{cg})$

and $\eta \approx 5$ generally and $\arg C(j \omega_{cg}) \leq \approx 43^\circ$

$\eta' g(A - B, \eta') \alpha'^2 - \alpha' + g(A - B, \eta') = 0$

with $g(A - B, \eta') = \frac{1}{1 + \eta'} - \frac{A - B}{4.6 \eta'^2}$

and $\eta' \approx 10$ generally and $A - B \leq \approx 1.18 \text{ rad/dec}$
As we need

\[ \arg C_1(j\omega) + \arg C_{cp}(j\omega) + \arg C_{lp}(j\omega) + \arg C_F(j\omega) + \arg G(j\omega) = \arg \beta(j\omega_{cg}) + A \log \frac{\omega}{\omega_{cg}} \]

**parameter \( \alpha \) of the constant phase element is such that**

\[ \arg C_{cp}(j\omega_{cg}) = \arg \beta(j\omega_{cg}) - \arg G(j\omega_{cg}) - \arg C_1(j\omega_{cg}) - \arg C_F(j\omega_{cg}) \]

**parameter \( \alpha' \) of the logarithmic phase element is such that around \( \omega_{cg} \)

\[ \frac{d}{d \log \omega} \arg C_{lp}(j\omega) = (A - B') \log \frac{\omega}{\omega_{cg}} \]

with \( B' = \frac{d}{d \log \omega} \arg G(j\omega) + \frac{d}{d \log \omega} \arg C_1(j\omega) + \frac{d}{d \log \omega} \arg C_F(j\omega) \)
A discrete-time control-system design problem with the sampling period $T_s$ is transformed into a pseudo-continuous problem.

Taking into account the zero-order hold effects: 

$$G_0(z) = \left(1 - z^{-1}\right)G(s)$$

Achieving the bilinear variable change $z^{-1} = \frac{1-w}{1+w}$, $G_0(z)$ becomes $G(w)$.

With $w = j\nu$ ($\nu$ is the pseudo-continuous frequency), $\nu = \tan\left(\frac{\omega T_s}{2}\right)$

The open-loop $\beta(w)$ and controller $C(w)$ are designed in the pseudo-continuous time domain as in the continuous time domain.

Finally, achieving the inverse variable change $w = \frac{1-z^{-1}}{1+z^{-1}}$, $C(w)$ becomes $C(z)$. 

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**Specific problems (Anti-windup systems)**

- An inner loop feedbacks a part of the controller

\[ K(s) = K'(s) \left( 1 + \frac{\omega_1}{s} \right) \]

- Taking into account the model of the plant nonlinearity and the describing function method, \( K(s) \) is split so that the linear behavior of the new controller remains the same than before:

\[ K(s) = \frac{K_y(s)}{1 + K_u(s)} \]

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Let $G$ be a plant whose nominal transfer function is:

$$G_0(s) = G_{mp}(s) e^{-\tau s} \prod_{i=1}^{n_z} \left(1 - \frac{s}{z_i}\right)$$

where: $G_{mp}(s)$ denotes its minimum-phase part; $z_i$ is one of its $n_z$ RHP zeros; $\tau$ is a time-delay.

Using the previous definition of $\beta(s)$ and $C(j\omega) = \beta(j\omega)/G_0(j\omega)$ leads to an unstable controller with a predictive part $e^{+\tau s}$.

Thus, the definition of $\beta(s)$ needs to be modified:

$$\beta(s) = \beta_{mp}(s) e^{-\tau s} \prod_{i=1}^{n_z} \left(1 - \frac{s}{z_i}\right)$$

where $\beta_{mp}(s)$ is the transfer function defined initially.
Specific problems (unstable plants)

Let $G$ be a plant whose nominal transfer function is:

$$G_0(s) = \frac{G_s(s)}{\prod_{i=1}^{n_p} \left(1 - \frac{s}{p_i}\right)}$$

where: $G_s(s)$ denotes its stable part; $p_i$ is one of its $n_p$ RHP poles.

To ensure the closed-loop stability, the open-loop transfer function can be defined:

$$\beta(s) = \beta_s(s) \left(e^{-j\pi}\right)^{n_p} \prod_{i=1}^{n_p} \frac{\left(1 + \frac{s}{p_i}\right)}{\left(1 - \frac{s}{p_i}\right)}$$

where $\beta_s(s)$ is the transfer function defined initially.

The $e^{-j\pi}$ term permits the satisfaction of the Nyquist criterion.
Let $G$ be a plant whose nominal transfer function is:

$$G_0(s) = G_d(s) \prod_{i=1}^{n_z} \left( 1 + 2\zeta_{z_i} \frac{s}{\omega_{nz_i}} + \frac{s^2}{\omega_{nz_i}^2} \right) \prod_{i=1}^{n_p} \left( 1 + 2\zeta_{p_i} \frac{s}{\omega_{np_i}} + \frac{s^2}{\omega_{np_i}^2} \right)$$

where: $G_d(s)$ is the well damped part; $(\omega_{nz_i}, \zeta_{z_i})$ and $(\omega_{np_i}, \zeta_{p_i})$ define the $n_z + n_p$ numerator or denominator lightly damped modes.

- To avoid the cancellation of some plant lightly damped modes,
- To attenuate the peak value of sensitivity functions $CS$ and $SG$,
- To attenuate the sensitivity functions $T$ and $S$ at some frequencies,

using $n_N$ notch filters, the open-loop transfer function can be defined:

$$\beta(s) = \beta_d(s) \prod_{i=1}^{n_z} \left( 1 + 2\zeta_{z_i} \frac{s}{\omega_{nz_i}} + \frac{s^2}{\omega_{nz_i}^2} \right) \prod_{i=1}^{n_p} \left( 1 + 2\zeta_{p_i} \frac{s}{\omega_{np_i}} + \frac{s^2}{\omega_{np_i}^2} \right) \prod_{i=1}^{n_N} \left( 1 + 2\zeta_{N_i} \frac{s}{\omega_{nN_i}} + \frac{s^2}{\omega_{nN_i}^2} \right)$$

where $\beta_d(s)$ is the transfer function defined initially.
Specific problems (nonlinear plants)

The nonlinear plant

\[
\begin{align*}
\dot{x} &= g(x, u) \\
y &= h(x, u)
\end{align*}
\]

can be input-output linearized by feedback for its nominal value. Nevertheless, the system obtained remains nonlinear for other parametric states.

Then, this perturbed nonlinear system is replaced by a set of linear models obtained using:

• the describing function method
• the first order linearization method around the set of possible trim points \(x_0(t)\).

From this set,

• a nominal frequency response is selected
• a set of frequency uncertainty domains are defined.
Specific problems (time-varying plants)

Two types of plants are considered:

- time-varying plants with periodic coefficients
- time-varying plants with asymptotically constant coefficients

The plants are modelled using $s$ time-varying transfer functions:

$$G(s, t) = e^{-st} \int_{-\infty}^{\infty} h(t, \xi)e^{s\xi} d\xi$$

A time-varying controller is designed such that:

• its connection in serie with the plant provides a time invariant system with a transfer function $\beta(s)$
• it minimizes the stability degree variations at the time of plant perturbations
• frequency-domain constraints on sensitivity functions are fulfilled.
Frequency-domain design of robust control systems

Lecture #4: MIMO CRONE control and applications
Contents

1 - Robustness
   1.1-Stability degree and stability margins
   1.2-Robustness and stability margins

2 - CRONE Control Design
   2.1-First generation
   2.2-Second generation
   2.3-Third generation
   2.4-Specific problems
   2.5-MIMO systems
   2.6-Industrial applications

3 – QFT Design
   3.1-Principle and methodology
   3.2-Application examples

Website for downloading:

User's forum:
CRONE CSD approaches for $N$ by $N$ MIMO systems

Decentralized controller (multi-SISO)

- SISO CRONE CSD of each diagonal element of a plant transfer function matrix
- Gershgorin and small gains theorems are used to take into account the off-diagonal elements as an additional uncertainty on each diagonal element

Full MIMO controller

Optimization of a nominal open-loop diagonal transfer function matrix to minimize the stability margin variations of the perturbed diagonal elements of the closed-loop complementary sensitivity transfer function.
The design of each controller diagonal element $K_i(s)$ is designed by taking into account:

- the diagonal nominal transfer function $G_{ii0}(s)$
- the structured uncertainty computed from all possible values $G_{ii}(s)$
- the unstructured uncertainty computed from the off-diagonal elements $G_{ji}(s)$ with $1 \leq j \leq N$ and $j \neq I$

*Independent design of each $\beta_i(s)$ (thus of each $K_i(s)$)*

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Multi-SISO uncertainty frequency domain

Let a $N$ by $N$ MIMO plant be known by its nominal parametric state, and 3 others parametric states.

The frequency uncertainty domain taken into account at frequency $\omega_k$ is defined from the convex hull of the 4 perturbed frequency responses $G_{ii_n}(j\omega_k)$ ($0 \leq n \leq 3$) encircled by column Gershgorin circles with $r_{in}(\omega_k)$ radius

$$r_{in}(\omega_k) = \sum_{1 \leq j \leq N \text{ and } j \neq i} \left| G_{ji_n}(j\omega_k) \right|$$

The multiplicative uncertainty $\Delta G_{ii_n}(j\omega_k)$ has an unstructured part whose magnitude is related to

$$\sum_{1 \leq j \leq N \text{ and } j \neq i} \left| \frac{G_{ji_n}(j\omega_k)}{G_{ii_n}(j\omega_k)} \right|$$

Circles in the complex plane become Ellipsoids in Nichols chart
The full MIMO controller $K(s)$ has to ensure 4 objectives

- perfect nominal decoupling $G_0(s)K(s) = \beta_0(s) = \text{diag}[\beta_{0i}(s)]_{i=1}^{N}$
- robustness of diagonal element of $T(s) = [1+G(s)K(s)]^{-1}G(s)K(s)$ resonant peaks
- accuracy specification (low frequency)
- control effort specification (high frequency)

**Simultaneous design of each open-loop $\beta_i(s)$**
MIMO transfer functions

\[ Y = (I + GK)^{-1} GKY_{\text{ref}} = SGKY_{\text{ref}} = TY_{\text{ref}} \]
\[ Y = (I + GK)^{-1} D_y = SD_y \]
\[ Y = (I + GK)^{-1} GD_u = SGD_u \]
\[ Y = -(I + GK)^{-1} GKN_m = -TN_m \]

\[ U = (I + KG)^{-1} KY_{\text{ref}} = K(I + GK)^{-1} Y_{\text{ref}} = KSY_{\text{ref}} \]
\[ U = -KSD_y \]
\[ U = -(I + KG)^{-1} KGD_u = -KSGD_u \]
\[ U = -KSN_m \]

*KG and GK are different matrices*
Nominal open-loop transfer function matrix

Each nominal and diagonal element of \( \beta_0(s) \) is defined by 3\(^{\text{rd}}\) generation CRONE open-loop transfer function:

\[
\beta_{0i}(s) = K_i \left( \frac{\omega_{i-N_i^-}}{s} + 1 \right)^{n_{i1}} N_i^+ \prod_{k=-N_i^-}^{N_i^+} \left[ \frac{1 + s/\omega_{i_k+1}}{1 + s/\omega_{i_k}} \right]^{a_{ik}} \Re\{ \left( \frac{1 + s/\omega_{i_k+1}}{1 + s/\omega_{i_k}} \right)^{ib_{ik}} \}^{-q_{i_k}\text{sign}(b_{ik})} \left[ \frac{1}{(1 + s/\omega_{iN_i^+})^{n_{ih}}} \right]
\]

As controller \( K(s) \) is defined by \( K(s) = G_0^{-1}(s)\beta_0(s) \), the low and high frequency orders of \( \beta_0(s) \) are defined from rank conditions based on the degree of numerators and denominators of \( G_0^{-1}(s) \).

\( \beta_0(s) \) has also to take into right half-plane and/or lightly damped poles of:

- **column of** \( G_0^{-1}(s) \) **as additional zeros of** \( \beta_0(s) \)
- **row of** \( G_0(s) \) **as additional poles of** \( \beta_0(s) \)
Optimization of open-loop transfer function matrix parameters

\[ \beta_0(s) = \text{diag} [\beta_{0i}(s)]_{1 \leq i \leq N} \]

Simultaneously, the parameters of each open loop nominal transfer function \( \beta_{0i}(s) \) are optimized to:

- minimize a cost function based on the variations of resonant peak of diagonal elements of \( T(s) \):
  \[
  \sum_{i=1}^{N} \left( \frac{\max G \times M_{ri} - M_{r0i}}{M_{r0i}} \right)^2 + \left( \frac{\min G \times M_{ri} - M_{r0i}}{M_{r0i}} \right)^2
  \]

- fulfil a set of shaping constraints on the diagonal and off-diagonal elements of closed loop transfer functions matrix \( T(s), S(s), CS(s) \) and \( SG(s) \).
Multi-SISO or MIMO approach

Multi-SISO CRONE CSD methodology (decentralized controller):

easy to use (independent design of each loop)

but provides only a diagonal controller

MIMO CRONE CSD methodology (full MIMO controller):

decoupling approach but many parameters to be optimized at the same time

Depending on a coupling analysis of the plant, multi-SISO or full-MIMO has to be chosen to optimize the ease-of-use/performance trade-off.
Analysing of coupling and of diagonal dominance

RGA analysis

Relative Gain Array (RGA) technique shows if the relationship between an input and an output could be modified by the other inputs. It shows the coupling level of a system and if a decentralized control could be efficient.

CD³ analysis

Taking account column plant elements, based on Gershgorin technique, an analysis of the Column Diagonal Dominance Degree (CD³) shows if a simple multi-SISO design could be used.
Robust RGA analysis

When along the desired closed-loop bandwidth, $\gamma_{ii}(j\omega)$ are close to 1 for all $G_n(s)$, a decentralized controller can be used ($u_i$ could be the good input to control output $y_i$ (as the relationship between $u_i(j\omega)$ and $y_i(j\omega)$ is never modified a lot by other inputs $u_n(j\omega)$ that ensure that output $y_j(j\omega) = 0$)

\[
\begin{bmatrix}
  y_1(j\omega) \\
  \vdots \\
  y_N(j\omega)
\end{bmatrix} = G(j\omega) \begin{bmatrix}
  u_1(j\omega) \\
  \vdots \\
  u_N(j\omega)
\end{bmatrix}
\]

$\Gamma(G_n(j\omega_k)) = G_n(j\omega_k) \times \left(G_n^{-1}(j\omega_k)\right)^T = [\gamma_{ij}(j\omega_k)]$ with $G_n \in G$

$\times$ Shur product (element-wise product)
$T$ transposed matrix

open-loop gain

with $\gamma_{ij}(j\omega_k) = \frac{y_i(j\omega_k)}{u_j(j\omega_k)}$ | $u_n(j\omega_k) = 0$ for $n \neq j$

closed-loop gain

$y_i(j\omega_k)$ | $y_n(j\omega_k) = 0$ for $n \neq i$
Robust CD\(^3\) analysis

\[
\begin{bmatrix}
y_1(j\omega) \\
\vdots \\
y_N(j\omega)
\end{bmatrix} =
\begin{bmatrix}
g_{11}(j\omega) & \cdots & g_{1N}(j\omega) \\
\vdots & \ddots & \vdots \\
g_{N1}(j\omega) & \cdots & g_{NN}(j\omega)
\end{bmatrix}
\begin{bmatrix}
u_1(j\omega) \\
\vdots \\
u_n(j\omega)
\end{bmatrix}
\]

The worst-case size of the unstructured multiplicative uncertainty that needs to be taken into account to design a decentralized controller with the multi-SISO approach is given by the coupling degrees

\[
\tau_{ci}(\omega_k) = \max_{G_n \in G} \left\{ \sum_{1 \leq j \leq N \text{ and } j \neq i} \left| \frac{G_{ji_n}(j\omega_k)}{G_{ii_n}(j\omega_k)} \right| \right\} \text{ for } 1 \leq i \leq N
\]

When within the desired closed-loop bandwidth all \(\tau_{ci}(\omega)\) are much smaller than 1, a decentralized controller can be designed with the multi-SISO approach (as the ellipsoids that model the unstructured uncertainty domain are small).
RGA & CD\textsuperscript{3} analysis examples

\[
G = \begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix} \Rightarrow \begin{cases} \Gamma = \begin{bmatrix} 1.01 & -0.01 \\ -0.01 & 1.01 \end{bmatrix} \\ \max \tau_{ci} = 0.01 \end{cases}
\]

Decentralized controller can be used

Multi-SISO design can be used

\[
G = \begin{bmatrix} 1 & 0.1 \\ 1 & 1 \end{bmatrix} \Rightarrow \begin{cases} \Gamma = \begin{bmatrix} 0.91 & 0.09 \\ 0.09 & 0.91 \end{bmatrix} \\ \max \tau_{ci} = 1 \end{cases}
\]

Decentralized controller can be used

MIMO design needs to be used

\[
G = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \Rightarrow \begin{cases} \Gamma = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \\ \max \tau_{ci} = 1 \end{cases}
\]

Decentralized controller cannot be used

MIMO design needs to be used
Decentralized or Centralized Controller? Multi-SISO or MIMO Design?

**RGA Analysis**
- Some $\gamma_{ii}(\omega)$ far from 1
- All $\gamma_{ii}(\omega)$ close to 1

**CD³ Analysis**
- Some $\tau_{ci}(\omega)$ non-smaller than 1
- All $\tau_{ci}(\omega)$ smaller than 1

**MIMO Design of a Centralized Controller**

**MIMO Design* of a Decentralized Controller**

**Multi-SISO Design of a Decentralized Controller**

* MIMO CSD with a simplified diagonal $G_0$
A 2x2 MIMO perturbed plant $G$ is defined by:

$$G(s) = \begin{bmatrix}
\frac{10}{(1+s)(1-s/\omega_n)} & \frac{k(1-s/100)}{(1+s/10)(1+s)(1-s/\omega_n)} \\
\frac{1}{(1+s)s} & \frac{5k(1-s/100)}{(1+s/10)s}
\end{bmatrix}$$

with $0.007 \leq \omega_n \leq 0.014$, $1/2 \leq k \leq 2$.

Nominal model $G_0$ of the plant is defined by:

$$\omega_n = 0.01 \text{ rad/s and } k = 1.$$
MIMO CRONE Control application (2/8)

The inverse $P(s)$ of the nominal transfer function matrix $G_0(s)$ is:

$$P(s) = \begin{bmatrix}
-10(s - 0.01)(1 + s)^2 & -0.02s(s + 1) \\
\frac{(s + 0.98)}{(s - 100)(s + 0.98)} & \frac{(s + 0.98)}{(s - 100)(s + 0.98)}
\end{bmatrix}$$

The nominal open-loop is defined by:

$$\beta_0(s) = \begin{bmatrix} \beta_{011}(s) & 0 \\
0 & \beta_{022}(s) \end{bmatrix}$$

- Taking into account the rows of $G_0(s)$, $SG(s)$ is stable if $s = +0.01$ is a pole of $\beta_{011}(s)$.
- Taking into account the columns of $P(s)$, $KS(s)$ is stable if $s = +100$ is a zero of $\beta_{011}(s)$ and of $\beta_{022}(s)$.
- As $G_0(s)$ has a pole $\omega_n$ in the 1st row much smaller than the required bandwidth, and an integration in the 2nd row, the control system will provide accuracy quickly if low-frequency orders $n_{11} > 1$ for $\beta_{011}(s)$ and $n_{12} > 1$ for $\beta_{022}(s)$.
- Controller $K(s)$ is proper if high-frequency orders $n_{h1} \geq 3$ for $\beta_{011}(s)$ and $n_{h2} \geq 2$ for $\beta_{022}(s)$.
- $N = 1$ for $\beta_{022}(s)$ to improve its optimization.
MIMO CRONE Control application (3/8)

For a strictly proper controller, the open-loop elements that have to be tuned can be defined by:

\[
\beta_{011}(s) = -K_1 \left( \frac{\omega_{10}}{s} + 1 \right)^2 \left( \frac{1 + s/\omega_{11}}{1 + s/\omega_{10}} \right)^{a_{10}} \left\{ \Re e/i \left( \frac{\alpha_{10}}{1 + s/\omega_{10}} \right) ib_{10} \right\}^{-q_{10}\text{sign}(b_{q10})} \left( \frac{1 + s}{0.01} \right) \left( \frac{1 - s}{100} \right) \left( \frac{1 - s}{0.01} \right) \left( \frac{s}{\omega_{11} + 1} \right)^4
\]

\[
\beta_{022}(s) = K_2 \left( \frac{\omega_{21}}{s} + 1 \right)^2 \left( \frac{1 + s/\omega_{20}}{1 + s/\omega_{21}} \right)^{a_{20}} \left\{ \Re e/i \left( \frac{\alpha_{20}}{1 + s/\omega_{20}} \right) ib_{20} \right\}^{-q_{20}\text{sign}(b_{q20})} \left( \frac{1 - s}{100} \right) \left( \frac{s}{\omega_{21} + 1} \right)^3 .
\]
For the $y_i/y_{ref_i}$ closed-loops:

- the desired bandwidth (which defines the corner frequency of $T_{u_{ii}}, S_{u_{ii}}$ and $SG_{u_{ii}}$) is respectively 1rad/s and 7rad/s
- the required resonant peaks $M_{rdi}$ are respectively 1dB and 0.7dB
- the resonant peak limitation ($T_{u_{ii}}$) is 4.5dB
- the sensitivity function limitation ($S_{u_{ii}}$) is 6dB
- the plant input sensitivity function limitation ($SG_{u_{ii}}$) is -20dB for $SG_{u_{i1}}$ and 0dB for $SG_{u_{i2}}$
- the control effort sensitivity function limitation ($KS_{u_{ii}}$) is respectively 50dB and 20dB for $KS_{i1}$ and $KS_{i2}$.
\[ \beta_{ii\text{ - eq}}(j\omega) = \frac{T_{ii}(j\omega)}{1 - T_{ii}(j\omega)} \]

- \( Y_{r1} = 7 \) dB, \( \omega_{r1} = 1 \) rad/s, \( \omega_{1\ 0} = 0.1 \) rad/s, \( \omega_{1\ 1} = 40 \) rad/s. Thus, \( a_{1\ 0} = 1.184 \), \( b_{q1\ 0} = -0.088 \), \( q_{1\ 0} = 1 \) and \( K_1 = 10.047 \). Cost function is 0.4 dB and all sensitivity constraints are met.

- \( Y_{r2} = 8.3 \) dB, \( \omega_{r2} = 5 \) rad/s, \( \omega_{2\ -1} = 0.1 \) rad/s, \( \omega_{2\ 0} = 2.5 \) rad/s, \( \omega_{2\ 1} = 80 \) rad/s. Thus, \( a_{2\ -1} = 1.23 \), \( b_{q2\ -1} = 0.3 \), \( q_{2\ -1} = 1 \), \( a_{2\ 0} = 1.156 \), \( b_{q2\ 0} = 0.463 \), \( q_{2\ 0} = 1 \) and \( K_2 = 2.232 \). Cost function is 0.6 dB and all sensitivity constraints are met.

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MIMO CRONE Control application (6/8)

\[ K_{R11}(s) = \frac{8024.89(s - 100)(s + 1.51)(s + 0.15)(s + 0.008)}{s^2(s + 64.3)(s + 47)(s + 14.2)} \]
\[ K_{R12}(s) = \frac{17682.14(s + 11.2)(s + 1.54)(s + 0.131)(s + 0.0099)}{s^2(s + 59.8)(s + 45.7)(s + 18.4)(s + 1.06)} \]
\[ K_{R21}(s) = \frac{39.08(s - 100)(s + 7.18)(s + 1.21)(s + 0.155)}{s(s + 141)(s + 124)(s + 5.18)(s + 0.766)} \]
\[ K_{R22}(s) = \frac{3604.44(s + 25.3)(s + 9.31)(s + 3.86)(s + 0.17)}{s(s + 169)(s + 72.5)(s + 35.1)(s + 1.76)} \]
MIMO CRONE Control application (7/8)
MIMO CRONE Control application (8/8)
First historic application of CRONE control principle
Control of the laser frequency by desensitizing the resonance peaks to
- temperature
- pressure
- cavity and amplification cell vibration
- dye inhomogeneity and turbulence.

Around an open loop unit gain frequency of 20kHz, a -3π/4 phase locking has been synthesised using a two way order 3/2 analog controller
Design of a PLL frequency demodulator (1979)

- Desensitization of the PLL demodulator stability degree to
  - ✓ reference signal input
  - ✓ reception level

- Use of an order ½ filter to ensure an order 3/2 open-loop behavior
Saffron picker wire-guided mobile robot (1987)

- Spanning cart guided by a conductor wire buried in the middle of each row:
  - 2 driving wheels at the rear
  - Electromagnetic signal emitted at the rear
  - Signal received and reemitted by the wire
  - Distance between the robot and the wire provided (with a nonlinear and badly known function) by 2 receiving coils in the front

- Desensitization of the stability degree of the control using a variable phase controller (2nd generation CRONE methodology) achieved with 7 zero/pole pairs.

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Robust control of the tool working depth, while limiting the slipping rate by taking into account:

- working depth of the tool
- nature of the tool (plough, decompactor, etc.)
- non-homogeneity and soil type (light, heavy, slipping).
- nonlinear static and dynamic model

Design of a 3rd generation CRONE controller, the reference depth being delivered by an outer loop supervising the slipping rate.

Tractor electro-hydraulic hitch-system (1991)

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Cutting tables for the textile industry (1992)

- Control of the XY displacement of a tool cutting leather, cloth, airbag, etc.
- Flexible belts taken into account
- Robustness with respect to the cutting tool used (laser, high pressure water, knife)
- Robustness with respect to the material (features and thickness) to be cut
- Increase of bandwidth (reduction of cutting time)
- Use of the 2\textsuperscript{nd} and then 3\textsuperscript{rd} generation CRONE control methodologies
Motor-pump group of a nuclear submarine (1994)

- Replacement of a switched controller (low speed/high speed) by one linear and robust controller
- System nonlinearities and perturbation on supply voltage taken into account by frequency-domain uncertainty domains
- 3rd generation CRONE controller designed for robustness and to manage control effort
- Smooth control from starting to operating speed and reduced settling time.
Car suspension (since 1995)

- Design of suspensions to manage as well as possible the comfort/handling trade-off and the robustness to mass-load uncertainty
- The 3 generation CRONE methodologies used to design passive and semi active new suspension devices
- Prototypes implemented on Citroën and Peugeot models (BX, XM, Xantia, 406, etc.)
Control of an lightly damped electromechanical chain (1999)

- Speed control of an electro-mechanical system with bending modes
- Nonlinear plant with lightly damped modes and load variations
- Use of an observer to estimate the load speed that have to be controlled
- Improvement of settling time
- Robustness with respect to uncertainty
- Use of the 3rd generation CRONE control methodology

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Skin pass rolling device (2004)

- Robust regulation of the speed of the cylinders without the direct measure of these speeds
- Same approach as that used for the previous Alstom system
- High quality cold rolled steel produced and sold to a Japanese car maker

![Diagram of skin pass rolling device with labels and symbols]

CRONE
Controller

CRONE
Controller

Power
Converter

Power
Converter

Motor shaft sup

Motor shaft inf

Spindle sup

Spindle inf

Motor inf

Back-up roll sup

Back-up roll inf

Work roll sup

Work roll inf

Observer

Observer

$\Omega^*_L$

$\Omega^*_M$

$\Omega^*_S$

$\Omega^*_M$

$\Omega^*_S$

$\Omega^*_L$

$\Gamma^*_M$

$\Gamma^*_S$

$\Gamma^*_L$

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Pneumatic actuated throttle plates (2001)

- Control of the opening angle of the valves
- Highly nonlinear plant
- Robustness of performance with respect to supply pressure variations (gain uncertainty) and user adjustment, etc.
- Use of the 2nd generation CRONE control methodology
- Reference signal provided by a fractional and nonlinear prefilter
- Performance improvements (static accuracy, bandwidth, etc.)
Active suspension system for vibration isolation (2002)

- Set of over demanding requirements for a reduced-order digital controller
- Design, and then closed-loop optimization, of a 1\textsuperscript{st} generation CRONE controller to reduce the magnitude spectrum of the residual disturbance
- The more difficult requirement overpassed only 0.33dB with a controller needing only 13 multiplications and 12 additions at each sampling period.
• Robust and high performing positioning of throttle plate actuators (Bosch, Delphi, Pierburg) for internal combustion engines

  ✓ Reduction of tuning time (from 2 months to half a day)

  ✓ Reduction of CPU load (increase of sampling time) and memory space (simplification of the controller)
Air-system control for combustion engines (2009)

- Model-based control of air-system (Gazoline and Diesel) to make vehicles cleaner
  - System identification for a set of torque/speed operating points
  - Parameterization of a Matlab-Simulink model
  - Design of robust multi-SISO and MIMO controllers

![Diagram of air-system control](image)
Air-system control for turbocharged Diesel engine (2012)

- Parameterization of a AMESim model
- Design of robust MIMO controllers
Pollutant Reduction of a Turbocharged Diesel Engine (2013)

- Design of a non square decentralized MIMO CRONE controller
- Reduction in both fuel consumption and NOx emission ($\approx -4\%$)
Modelling and control of sailboats (2013)

• Improvement of automatic pilot using CRONE robust controller
• Robustness with respect to sailing operating conditions and boats
Control of wind turbine (2010)

- MMPT CRONE control on a new wind turbine design

- Mechanical load reduction using CRONE controllers
Anti-icing & de-icing system for wind turbine blade (2012)

- A polymer and electrically conductive paint is used for de-icing and anti-icing
- The temperature of the paint is controlled by a CRONE controller (various thermal behavior on the blade)
- Paint temperature is measured by PT100 probes
- Tested in a wind tunnel and outdoor on a blade prototype
• Modelling and system identification of a MIMO system
• Design of robust full-MIMO controllers for speed and torque tracking
Modeling and robust control of a kite in dynamic flight (2015)

- Modeling of a kite with a dynamic flight
- Robust control of the kite to extract the maximal energy from wind
- Application to the traction of ships
Enjoy now!  


The CRONE (R) Toolbox, developed by the CRONE research group, is a Matlab and Simulink Toolbox dedicated to fractional calculus. The original theoretical and mathematical concepts, developed in the group, are used in the toolbox. During the last ten years, the CRONE Toolbox has been used by industrial partners in many applications.

The last two decades have witnessed a growing interest in fractional derivatives and their applications. The aim of the CRONE group is to share its developments and its knowledge with scientists, researchers, and engineers worldwide.

For any specific developments, the CRONE group welcomes all collaborations.

A classical version of the toolbox has a Guided User Interface (GUI) and contains three main modules:
- The "Mathematical module" implements several algorithms for fractional calculus.
- The "system-identification module" extends the common system-identification frequency and time domain tools to fractional order models.
- The "CRONE control" module uses fractional orders as high-level design parameters in order to make easy the design of robust control-systems.

An object oriented version of the toolbox is currently being developed. It contains multiple scripts and allows many enhancements related to object oriented programming:
- Overloading of basic operators $(+, -, *, /, \ldots)$
- Overloading of standard Matlab scripts (lsim, bode, Nichols, ...) for the new classes.

As a consequence, an end user familiar with standard Matlab operators and scripts can use straightforwardly the CRONE toolbox.

Both classical and object oriented CRONE Toolboxes can be downloaded.

Welcome aboard the CRONE Toolbox Airlines.
Development of a MIMO CRONE Toolbox

PID design(s)

CRONE SISO design(s)

CRONE MIMO design(s)
UE E9AM2AS-C AU306
Frequency-domain design of robust control-systems

Lecture#5: QFT design
Contents

1 - Robustness
  1.1-Stability degree and stability margins
  1.2-Robustness and stability margins

2 - CRONE Control Design
  2.1-First generation
  2.2-Second generation
  2.3-Third generation
  2.4-Specific problems
  2.5-MIMO systems
  2.6-Industrial applications

Website for downloading:

User's forum:

3 – QFT Design
  3.1-Principle and methodology
  3.2-Application examples
QFT – Quantitative Feedback Theory

- Developed by Horowitz, Sidi et al. since the 60’s and inspired by the works of Bode (30-40’s).
- Frequency domain technique using the Nichols chart in order to achieve a robust control-system for a perturbed plant with a given uncertainty.
- Toolboxes have been developed by Borghesani/Chait/Yaniv (Terasoft), Garcia-Sanz/Philippe/Houpis (Case Western Reserve University), etc.
- Methodology often underestimated by the proponents of «modern control theory» (State Space, LQR, $H_\infty$, etc.).
Quantitative is related to the specifications (desired performance) that must be defined quantitatively: acceptable overshoot for tracking response or disturbance rejection, acceptable settling times, etc.

For a perturbed/nonlinear/LTV plant $P$, the objective of a QFT design is to determine the LTI feedback controller $G(s)$ and the LTI prefilter $F(s)$ that ensure:

$$T_D \in T_D, \ T_R \in T_R, \ etc.$$ for all $P \in P$,

where $T_D$ is a set of the acceptable transfer functions $Y(s)/D(s)$, $T_R$ a set of the acceptable transfer functions $Y(s)/R(s)$ and $P$ the set of all possible (LTI) transfer function model $P(s)$ of the plant $P$. 

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QFT is a graphical methodology whose main part will make you an adept of the Nichols chart.

The design of a QFT control-system requires 5 main steps.

1. From a set of time-domain specifications (for tracking, disturbance rejection, control effort, etc.), frequency domain bounds are defined to constrain the closed loop transfer functions.

2. Templates that model the frequency uncertainty domains of the plant are computed for a set of well chosen frequencies values.

3. From the closed loop frequency constraints and from the templates, boundaries related to an arbitrary selected nominal open loop transfer function are constructed on the Nichols chart.
4. Taking into account the boundaries, the nominal open-loop (product of the nominal plant \( P_0(s) \) and of the controller \( G(s) \)) is iteratively shaped in order to:
   - ensure robust stability degrees
   - ensure robust disturbance rejections
   - manage the control efforts
   - ensure sufficiently small variations of the tracking response.

5. Controller \( G(s) \) ensuring small variations of the tracking response \( Y(s)/R(s) \), prefilter \( F(s) \) is then iteratively defined in order that all Bode plots of \( Y(s)/R(s) \) belong to \( T_R \) (set of the acceptable tracking transfer responses).
Step 1 – Tracking specification

For all $P \in \mathcal{P}$, the response of the output $y(t)$ to a reference input $r(t)$ step variation must lie between specified upper $y_U(t)$ and lower $y_L(t)$ bounds. These extreme responses can be characterized by rising and settling times, peak time and peak overshoot.

$y_U(t)$ upper output response; $y_L(t)$ lower output response;
$t_R$ rising time; $t_p$ and $M_p$ peak time and overshoot
$\delta_R(\omega_i)$: acceptable tolerance between $|T_{RL}(j\omega)|_{dB}$ and $|T_{RU}(j\omega)|_{dB}$

Upper and lower frequency response of $T_{RU}(s)$ and $T_{RL}(s)$ can be deduced from the Fourier transform of $y_U(t)$, $y_L(t)$ and $r(t)$. As for all $P \in \mathcal{P}$, $Y(j\omega)/R(j\omega)$ must lie between $T_{RU}(j\omega)$ and $T_{RL}(j\omega)$ bounds, for all frequency $\omega_i$, controller $G$ needs to ensure an uncertainty of $|Y(j\omega)/R(j\omega)|_{dB}$ lower than $\delta_R(j\omega_i)$.  

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For a step output disturbance, one could wish that its effect on the plant output would be lower than a given value $\alpha$ before a given time $t_x$.

\[ \delta_D(\omega_i) : \text{desired attenuation of } d(j\omega) \text{ at } \omega_i (\text{upper bound of } |T_D(j\omega_i)|_{dB}) \]

Values of $t_x$ and $\alpha$ permit the determination of the greatest acceptable value $\delta_R(j\omega_i)$ of $|Y(j\omega_i)/D(j\omega_i)|_{dB}$ at frequency $\omega_i$.

The greatest value of overshoot $\alpha_p$ can be bounded by specifying the resonant peak of the complementary sensitivity function.
Frequency domain constraints on closed loop transfer functions

\[
\begin{align*}
\frac{PG}{1 + PG} & \leq W_1, \text{ constraint on the complementary sensitivity function } (Y/N) \\
\frac{1}{1 + PG} & \leq W_2, \text{ reduction of the sensitivity function } (Y/D) \\
\frac{P}{1 + PG} & \leq W_3, \text{ rejection of the input disturbance } (Y/V) \\
\frac{G}{1 + PG} & \leq W_4, \text{ limitation of the sensitivity of the plant input to noise } (U/N) \\
W_5 \leq & \frac{FPG}{1 + PG} \leq W_6, \text{ constraints on tracking response } (Y/R)
\end{align*}
\]
1. Arbitrary choice of a nominal LTI model of the plant
2. Definition of a small number of well chosen frequencies $\omega_i$ in a range around the required closed loop bandwidth
3. Computation of all possible frequency responses $P(\omega_i)$
4. Determination of the set templates: smallest hull $D_i$ that includes the frequency responses $P(\omega_i)$
For each kind of specification $W_{sk}$, and for the set of frequency $\{\omega_i\}$, a graphical frequency domain boundary $B_k$ is constructed on the Nichols chart. A boundary is constructed in order that

- when the nominal open loop frequency response (for $G(s)P(s) = G(s)P_0(s)$) respects it
- the closed loop bound $W_{sk}$ is robustly respected for all $P \in \mathcal{P}$

This construction is achieved from the $W_{sk}$ specification and the set of templates $\{D_i\}$.

For each frequency $\omega_i$ and for the set of specification $W_{sk}$ the most restrictive portions of $B_k$ are selected to construct the composite boundary $B_0$ that will be used to shape the nominal open loop frequency response.
Construction of a U-Contour

In order to limit the step response overshoot of $y$ to $d$ (and to $r$ with $F = 1$), the open loop frequency response need to avoid a $M_r \text{dB}$ contour of the Nichols chart for all $P \in P$. For vertical template (gain like uncertainty), this condition is met for all $\omega$ when the nominal point of the template remains outside of a U-contour.

Example:

- $M_r = 2.3 \text{dB}$
- HF ratio between $\max|P(j \omega)|$ and $\min |P(j \omega)| = 20 \text{dB}$.

If the nominal point correspond to the lowest magnitude of $P$, the U-contour is constructed from the upper part of the $M_r$ contour and from its lower part moved down of the length of the template.

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Construction of U-Contours for gain&phase templates

In order to use a controller with a phase lead effect as small as possible (to limit the control effort), a set of different U-contours need to be constructed for each frequency of the set \{ \omega_i \} by taking into account the genuine shape of each template \( D_i \).

The U-contour is then constructed by mixing the \( M_r \) dB contour and each template \( D_i \).
Construction of tracking boundaries $B_R$

For each $\omega_i$, the boundary $B_R(\omega_i)$ lies from the U-contour to the magnitude axis, and needs to ensure:

$$\max_{P \in P} |T(j \omega_i)|_{dB} - \min_{P \in P} |T(j \omega_i)|_{dB} \leq \delta_R(\omega_i)$$

For a set of open-loop phase values, one has to find for the magnitude of the nominal open-loop frequency response that ensures:

$$\max_{P \in P} |T(j \omega_i)|_{dB} - \min_{P \in P} |T(j \omega_i)|_{dB} = \delta_R(\omega_i)$$

Example: $P = \left\{ \frac{k}{(1 + \tau s)} : \begin{array}{l} k \in [1,10] \ \tau \in [1,10] \\ \end{array} \right\}$

- $\delta_R(0.01) = 1$ dB
- $\delta_R(1) = 30$ dB

Smaller is the tolerance $\delta_R(\omega_i)$ and larger is the template, higher needs to be the boundary $B_R(\omega_i)$. When the tolerance is too small, the controller will be an high gain controller.
The output disturbance rejection objective is defined by: \[ |S(j\omega_i)|_{dB} \leq \delta_D(\omega_i) \]

For a set of open-loop phase values, one has to find for the magnitude of the nominal open-loop frequency response that ensures:

\[ \max_{P \in P} |S(j\omega_i)|_{dB} = \delta_D(\omega_i) \]

Example: \( P(s) = \frac{k}{(1+\tau s)} \) with \( k \in [1,10], \tau \in [1,10] \)

- \( \delta_D(10^{-3}) = -19 \text{ dB} \)
- \( \delta_D(0.01) = -3 \text{ dB} \)

The chart that gives the set of open-loop points that leads to a constant magnitude of \( |S| \) is the reversed common Nichols chart (named «lochin contour» by D’Azzo and Houpis).

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For each $\omega_i$, the boundary $B_0(\omega_i)$ is constructed by selecting the most restrictive portions of each $B_k(\omega_i)$. For tracking and disturbance constraints, the composite boundary is defined by all the upper portions of $B_R(\omega_i)$ and $B_D(\omega_i)$. The composite boundary $B_0(\omega_i)$ is then used as the unique graphical constraint guiding the nominal open loop frequency response placement for $\omega = \omega_i$.

Example: $P(s) = \begin{cases} \frac{k}{(1 + \tau s)} & k \in [1,10], \ \tau \in [1,10] \\ \end{cases}$

- composite frequency for $\omega = 0.01 \text{ rad.s}^{-1}$
Depending of each kind of specification, several kinds of boundary are constructed:

- boundaries (solid line) for whom the open loop frequency point needs to be above
- boundaries (dashed line) for whom the open loop frequency point needs to be below

For a given frequency $\omega_i$, when a second kind of boundary is fully below a first kind one, the composite frequency does not exist and the initial specifications need to be modified (incompatible specifications).
Step 4 – Iterative loop shaping

For each frequency $\omega_i$, $L_0(j \omega_i) = P_0(j \omega_i)G(j \omega_i)$ needs to be placed in order to meet the composite boundary $B_0(\omega_i)$. The robust $L_0(s)$ is built up step by step by adding zero and pole terms.

Example:

$L_{01}(s) = K_1P_0(s)$

$L_{02}(s) = L_{01}(s)\frac{K_2}{s}$

$L_{03}(s) = L_{02}(s)(1 + \tau_1s)$

$L_{04}(s) = \frac{L_{03}(s)}{(1 + \tau_2s)}$

etc.
Step 5 – Design of the prefilter

As controller $G(s)$ ensures that

$$\max_{P \in \mathcal{P}} |T(j \omega_i)|_{dB} - \min_{P \in \mathcal{P}} |T(j \omega_i)|_{dB} \leq \delta_R(\omega_i)$$

it also ensures

$$\max_{P \in \mathcal{P}} |F(j \omega_i)T(j \omega_i)|_{dB} - \min_{P \in \mathcal{P}} |F(j \omega_i)T(j \omega_i)|_{dB} \leq \delta_R(\omega_i)$$

The tracking response $Y(s)/R(s)$ is then tuned by designing the prefilter $F(s)$ that ensures

$$|F(j \omega_i)|_{dB} + \max_{P \in \mathcal{P}} |T(j \omega_i)|_{dB} \leq |T_{RU}(j \omega_i)|_{dB}$$

$$|F(j \omega_i)|_{dB} + \min_{P \in \mathcal{P}} |T(j \omega_i)|_{dB} \geq |T_{RL}(j \omega_i)|_{dB}$$

$F(s)$ is built up step by step by adding zero and pole terms in order that all Bode plots of $Y(s)/R(s)$ lie between $T_{RU}$ and $T_{RL}$. 

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Application #1 (using the QFT toolbox)

\[ P = \left\{ P(s) = \frac{k}{(s + a)(s + b)} \right\} \text{ with } k \in [1, 10], a \in [1, 5], b \in [20, 30] \]

Specifications:

\[ \left| \frac{P(j\omega)G(j\omega)}{1 + P(j\omega)G(j\omega)} \right| \leq \mu = 1.2, \text{ for all } P \in \mathcal{P} \text{ and } \omega \in [0, \infty) \]

\[ \left| \frac{Y(j\omega)}{D(j\omega)} \right| \leq 0.02 \left(\frac{(j\omega)^3 + 64(j\omega)^2 + 748(j\omega) + 2400}{(j\omega)^2 + 14.4(j\omega) + 169}\right), \text{ for all } P \in \mathcal{P} \text{ and } \omega \in [0, 10] \]

\[ \left| \frac{Y(j\omega)}{V(j\omega)} \right| \leq 0.01, \text{ for all } P \in \mathcal{P} \text{ and } \omega \in [0, 50] \]
Construction of the plant templates

1. Gridding of all plant uncertain parameters and/or operating points
2. Arbitrary selection of a nominal plant model
3. Definition of a small number of frequencies around the required closed loop bandwidth
4. Plot of templates
Construction of Nichols chart bounds

1. Definition of the open-loop boundaries related to each closed-loop constraint

2. Merging of all boundaries

3. Definition of the composite boundaries to be taken into account
Iterative design of the controller (1/2)

Loop shaping with a graphical user interface to define the gain/zeros/poles of $G$

Proportional controller: $G = 379$
Iterative design of the controller (2/2)

Improper controller

\[ G(s) = 379 \left( \frac{s}{42} + 1 \right) \]

Biprop er controller

\[ G(s) = \frac{379 \left( \frac{s}{42} + 1 \right)}{\frac{s^2}{165} + 1} \]

Strictly proper controller

\[ G(s) = \frac{379 \left( \frac{s}{42} + 1 \right)}{\frac{s^2}{247^2} + \frac{s}{247} + 1} \]
Analysis of closed-loop frequency responses

Checking if all specifications are satisfied using a large set of frequencies

<table>
<thead>
<tr>
<th>Weight: W1 (T)</th>
<th>Weight: W2 (S)</th>
<th>Weight: W3 (PS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (rad/sec)</td>
<td>Magnitude (dB)</td>
<td>Frequency (rad/sec)</td>
</tr>
<tr>
<td>10^{-2} - 10^{-1}</td>
<td>0 - 10</td>
<td>10^{-2} - 10^{-1}</td>
</tr>
<tr>
<td>10^{-1} - 10^{0}</td>
<td>-10 - 20</td>
<td>10^{-1} - 10^{0}</td>
</tr>
<tr>
<td>10^{0} - 10^{1}</td>
<td>-20 - 30</td>
<td>10^{0} - 10^{1}</td>
</tr>
<tr>
<td>10^{1} - 10^{2}</td>
<td>-30 - 40</td>
<td>10^{1} - 10^{2}</td>
</tr>
<tr>
<td>10^{2} - 10^{3}</td>
<td>-40 - 50</td>
<td>10^{2} - 10^{3}</td>
</tr>
</tbody>
</table>

When one specification is not satisfied, the corresponding frequency can be added in the initial frequency set to improve the loop shaping.

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Design of a controller with a large bandwidth

\[ G(s) = \frac{8000}{1 + \frac{1}{s} + \frac{3790}{s^2 + \frac{1}{s^2}} + \frac{1}{s}} \]
Large bandwidth controller with a bounded control effort (1/2)

Additional constraint: \[ \left| \frac{G}{1 + PG} \right| \leq W_4 = 100 \text{dB} \]

\[ G(s) = \frac{2318 \left( \frac{1}{80}s + 1 \right) \left( \frac{1}{85}s + 1 \right)}{\left( \frac{1}{100}s + 1 \right) \left( \frac{1}{2000^2}s^2 + \frac{1}{2000}s + 1 \right)} \]
Large bandwidth controller with a bounded control effort (2/2)
Application #2 – Design of a prefilter

\[ P = \left\{ P(s) = \frac{ka}{s(s+a)} \text{ with } k \in [1,10], a \in [1,10] \right\} \]

Specifications:

\[ \left| \frac{P(j\omega)G(j\omega)}{1 + P(j\omega)G(j\omega)} \right| \leq \mu = 1.2 \text{ for all } P \in P \text{ and } \omega \in [0, \infty) \]

\[ T_L(\omega) \leq \left| F(j\omega) \frac{P(j\omega)G(j\omega)}{1 + P(j\omega)G(j\omega)} \right| \leq T_U(\omega) \text{ for all } P \in P \text{ and } \omega \in [0,10] \]

with

\[ T_L(\omega) = \left| \frac{0.06584(j\omega + 30)(j\omega + 10)}{(j\omega)^2 + 4(j\omega) + 19.742} \right| \]

\[ T_U(\omega) = \left| \frac{120}{(j\omega)^3 + 17(j\omega)^2 + 82(j\omega) + 120} \right| \]
Design of the feedback controller

Controller $G$ ensures that the variation of $T$ is smaller than the tolerance defined by $T_L$ and $T_U$.
Shaping of the prefilter

Use of the graphical user interface to define the gain/zeros/poles of $F$

$$F(s) = \frac{1}{4^2 s^2 + \frac{2 \times 0.7}{4} s + 1}$$
Mise en œuvre des méthodologies CRONE et QFT
(2x4h)

Objectif

L’objectif est de mettre en œuvre les méthodologies CRONE (de première, deuxième puis troisième génération) et QFT pour la synthèse de commandes robustes. Tout d’abord on mettra en évidence le problème de robustesse dû aux incertitudes paramétriques du procédé. Le premier procédé à asservir sera celui ayant déjà été utilisé pour l’étude d’un système de désaturation en 2ème année de l’ENSEIRB-MATMECA. Seule l’inertie initialement considérée minimale est maintenant augmentée de l’inertie incertaine de la charge. Ensuite, on considérera le procédé incertain utilisé en cours pour illustrer la commande CRONE de 3ème génération.

Outils


Description du premier procédé

Le procédé à commander est réalisé à partir d’un moteur de marque Parvex, à courant continu et à aimant permanent, pouvant fournir un couple nominal de 0.3Nm, et commandé en courant à travers un variateur comportant une boucle de courant et une limite de la vitesse de rotation et du courant d’induit du moteur. La commande se fait en -5V/5V et la position de l’arbre de sortie du moteur est fournie par un codeur incrémental 500 traits lu en mode quadruple.

Le modèle équivalent au procédé est représenté par le schéma synoptique donné par la figure 1 où $F(s)$ représente le signal équivalent à une perturbation de couple.

![Figure 1 - Modèle linéaire équivalent du procédé](image)

La fonction de transfert du procédé $G(s)$ reliant les variations de la position angulaire en radians, $Y(s)$, aux variations du signal de commande en volts, $U(s)$, que l’on pourra dans le domaine de fréquence considérée mettre sous la forme :

$$G(s) = \frac{G_0}{s^2(1+0.001s)},$$

où le gain $G_0$ dépend de l’inertie en rotation : $600 \leq G_0 \leq 4000$ et $G_{0\text{nom}} = 1550$. 
Cahier des charges de la loi de commande

On souhaite déterminer une loi de commande en boucle fermée qui assure :

- un temps de réponse $t_r$ aussi petit que possible ;
- un premier dépassement réduit de la réponse indicielle de l’ordre de 25% à 30% ;
- un rejet de toute perturbation d’entrée constante ;
- un bruit de commande inférieur à 10 fois le bruit de mesure HF ;
- un système en boucle ouverte aux THF.

1 – Synthèse d’une loi de commande de type PID(F)

Le cahier des charge peut être vérifié en utilisant un régulateur PID assurant une marge de phase de 50° avec une fonction de transfert :

$$K(s) = K_0 \frac{1 + \frac{s}{\omega_l}}{\frac{s}{\omega_i} \left(1 + \frac{s}{\omega_2}\right) \frac{1}{1 + \frac{s}{\omega_f}}}.$$  

Figure 2 – Boucle d’asservissement

Si l’on note $\omega_u$ la fréquence au gain unité en boucle ouverte, on pourra prendre $\omega = \frac{\omega_u}{5}$, $\omega_i = \frac{\omega_u}{\alpha}$, $\omega_2 = \alpha \omega_u$ et $\omega_r = 5 \omega_u$. Tout d’abord, déterminer $\alpha$ pour assurer la marge de phase en présence du procédé nominal. En écrivant la condition de gain relative à la boucle ouverte à la fréquence $\omega_u$, on montre que la condition portant sur le gain HF du régulateur conduit à :

$$\omega_u^2 \approx \frac{10G_{0
om}}{\alpha}$$  

Déterminer $\omega_u$, $\omega_i$, $\omega_r$, $\omega_2$ puis $K_0$.

A travers les variations de la marge de phase, du facteur de résonance et du premier dépassement de la réponse indicielle, analyser la robustesse de cette commande.

2 – Utilisation de la méthodologie CRONE de 1ère génération

Synthétiser une loi de commande permettant de respecter le cahier des charges pour tous les états du procédé. Le régulateur est de la forme :

$$K(s) = K_0 \frac{1 + \frac{s}{\omega_l}}{\frac{s}{\omega_i} \left(1 + \frac{s}{\omega_2}\right)^n \frac{1}{1 + \frac{s}{\omega_f}}}.$$
Déterminer tout d'abord le rapport \( \omega_h/\omega_l \) nécessaire pour assurer la robustesse de la commande. Pour simplifier, on pourra prendre \( \omega_l = \omega_i \) et \( \omega_f = \omega_h \). Après quelques itérations, déterminer le régulateur conduisant à la plus grande valeur possible de la fréquence \( \omega_{cg} \).

Déterminer l'approximation rationnelle du régulateur et vérifier la robustesse de cette commande.

### 3 – Mise en défaut de la commande CRONE de 1ère génération

On permet maintenant un bruit de commande 10 fois plus grand que précédemment.
Déterminer le nouveau régulateur CRONE de 1ère génération assurant la plus grande fréquence \( \omega_{cg} \) possible.

Analyser la robustesse de cette commande.

### 4 – Utilisation de la méthodologie CRONE de 2ème génération

En utilisant la méthodologie CRONE de 2ème génération fondée sur une fonction de transfert en boucle ouverte de la forme

\[
\beta(s) = K \left( \frac{1 + s/\omega_l}{s/\omega_l} \right)^{n_l} \left( \frac{1 + s/\omega_h}{1 + s/\omega_l} \right)^{n_h} \left( \frac{1}{1 + s/\omega_h} \right),
\]

après quelques itérations, synthétiser la loi de commande robuste et conduisant à la plus grande valeur possible de la fréquence \( \omega_{cg} \).

Déterminer le régulateur rationnel conduisant à la boucle ouverte fractionnaire obtenue pour l'état nominal du procédé, puis vérifier la robustesse de cette commande.

### Description du second procédé

Le procédé est représenté par le schéma synoptique donné par la figure 3 où \( V(s) \) et \( D(s) \) représentent des perturbations d'entrée et de sortie de type échelon.

![Figure 3 - Modèle du procédé](image-url)

Le modèle du procédé à asservir est défini par la fonction de transfert

\[
G(s) = \frac{10}{s(1+10s)} \left( \frac{1 + s}{\omega_p} \right) \left( \frac{1 + s/10}{1 + s/100} \right).
\]
Les valeurs nominales des paramètres sont $\tau_0 = 1000s$, $\omega_{z0} = 50\text{ rad/s}$ et $\omega_{p0} = 1\text{ rad/s}$.

**Cahier des charges des lois de commande**

On souhaite déterminer des lois de commande qui assurent :

- un maximum de la fonction de sensibilité complémentaire inférieur à 2.5 dB ;
- une marge de module supérieure à 0.5 ;
- un temps de rejet, $t_0$, de la perturbation de sortie inférieur à 2.5 s ;
- un rejet de toute perturbation d'entrée constante avec un temps de rejet aussi rapide que possible ;
- une amplification du bruit de mesure HF vers le signal de commande inférieure à 140 dB ;
- un système en boucle ouverte aux THF.

5 – Utilisation de la méthodologie CRONE de 3ième génération

Maintenant les deux premières générations de la méthodologie CRONE prises en main, on se propose de découvrir celle de troisième génération. Le travail déterminant consistera en la définition des contraintes portant sur les fonctions de sensibilité et à l'optimisation des performances. L'analyse critique des résultats obtenus sera particulièrement appréciée.

6 – Mise en œuvre de la méthodologie QFT

L'objectif est ici la mise en œuvre de la méthodologie QFT à l'aide de la boîte à outils QFT de Matlab. Le procédé asservi étant le même que celui utilisé pour l'application de la commande CRONE de troisième génération, on pourra comparer ces deux approches, à travers d'une part leurs résultats et leur facilité de mise en œuvre d'autre part.

6.1 – Synthèse d'une loi de commande à 1 degré de liberté

On adopte le schéma de commande de la figure 4.

**Figure 4 – Boucle d'asservissement**

1.1 – Déterminer l'ensemble des contraintes fréquentielles permettant de respecter l'ensemble du cahier des charges.

1.2 – Sachant que $\pi /3 \leq \tau \leq \pi * 3 \omega_e = \omega_{z0}$ et $\omega_p = \omega_{p0}$, définir l'ensemble des fonctions de transfert possibles puis visualiser les domaines d'incertitudes. On pourra prendre $[0.01, 0.1, 1, 2, 5, 10, 100]$ comme vecteur initial des fréquences significatives de ce problème de commande.

1.3 – Définir chacune des contraintes. Observer les zones du plan de Nichols à respecter.

1.4 – Utiliser l'environnement graphique pour déterminer le régulateur $G(s)$ permettant de respecter le cahier des charges.
1.5 – Vérifier le respect du cahier des charges sur toute la gamme de fréquence nécessaire. Si ce respect n'est pas parfait, intégrer la (ou les) fréquence en cause dans le vecteur des fréquences significatives et réitérer les étapes 1.2 à 1.5.

A travers les variations de la marge de phase, du facteur de résonance et du premier dépassement de la réponse indicielle, analyser la robustesse de cette commande.

6.2 – Synthèse d'une loi de commande à 2 degrés de liberté

On adopte le schéma de commande de la figure 5.

![Figure 5 – Boucle d’asservissement et préfiltre](image)

Au cahier des charges précédent, on ajoute les contraintes suivantes :

- la fonction de transfert relative au rejet de la perturbation d'entrée doit être limitée en amplitude par celle du transfert \( s/(0.6+30s) \).
- la réponse indicielle à la consigne doit se situer entre celles de la fonction de transfert :

\[
T_Y(s) = \frac{T_{Y0}}{1 + 1.4 \frac{s}{\omega_h} + \left(\frac{s}{\omega_h}\right)^2} \left(1 + \frac{s}{10}\right)^3 \text{ avec } (T_{Y0}, \omega_h) \text{ défini par } (0.9, \ 0.1 \text{ rad/s}) \text{ et } (1.1, 0.4 \text{ rad/s}).
\]

2.1 – Déterminer l’ensemble des contraintes fréquentielles permettant de respecter l’ensemble du cahier des charges.

2.2 – Sachant que \( \tau = \tau_0 \omega_b = \omega_{b0} \) et \( \omega_b/3 \leq \omega_b \leq \omega_{b0} \cdot 3 \), déterminer l’ensemble des fonctions de transfert possibles. On pourra prendre \([0.01, 0.1, 1, 2, 5, 10, 100]\) comme vecteur initial des fréquences significatives de ce problème de commande.

2.3 – Définir les zones du diagramme de Nichols à respecter par la fonction de transfert en boucle ouverte.

2.4 – Déterminer le régulateur \( G(s) \) permettant de respecter le cahier des charges.

2.5 – Vérifier le respect du cahier des charges sur toute la gamme de fréquence nécessaire.

2.6 – A l’aide de l’environnement graphique, déterminer le préfiltre \( F(s) \) permettant de respecter le cahier des charges relatif au suivi de consigne.

Vérifier la robustesse de cette commande.